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THE BOND-GRAPH MODEL OF A VIBRATING DAMPED SYSTEM PROVIDED WITH A DYNAMIC ABSORBER WITH VISCOUS FRICTION

BY

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Abstract. The vibrations coming from different exterior sources generally cause undesirable effects on mechanical systems, leading to important malfunctions in their behaviour. The mitigation of these negative effects of vibrations could be done in different manners, one of them being the attachment of a vibration dynamic absorber. The work presents the mathematical model of a system subjected to a sinusoidal force, provided with a dynamic absorber with viscous friction, obtained by using the bond-graph model. The bond-graph model generates the state equations which are integrated in MATLAB-SIMULINK.

Keywords: dynamic absorber; bond-graph; state-space equations; SIMULINK; system dynamics.

1. Introduction

Exterior variable forces could act upon mechanical systems and they produce vibrations that disturb the normal behaviour of the system. The effects of these undesirable vibrations, which occur due to the interaction between the

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mechanical system and the environment, should be minimized (Elias and Matsagar, 2017). One of the possible solutions is to attach to the disturbed system a device called vibration dynamic absorber (Rao, 2018). Its role is to diminish the amplitude of the vibrations induced by the oscillatory forces coming from the system environment. There are several types of vibration dynamic absorbers. The present work approaches the mathematical modelling of a dynamic absorber with viscous damping, attached to a mechanical system subjected to the action of an exterior, disturbing force with sinusoidal variation. The mathematical model offers the possibility of a numerical simulation, needed in the adoption of optimum values for the dynamic absorber parameters.

This approach is very useful when it is difficult to find an analytical solution of the differential equations system (Pană, 1984).

2. Description of the Mechanical System

The mechanical system has two component subsystems. The first subsystem consists of a spring of elastic constant k and a damper, having the

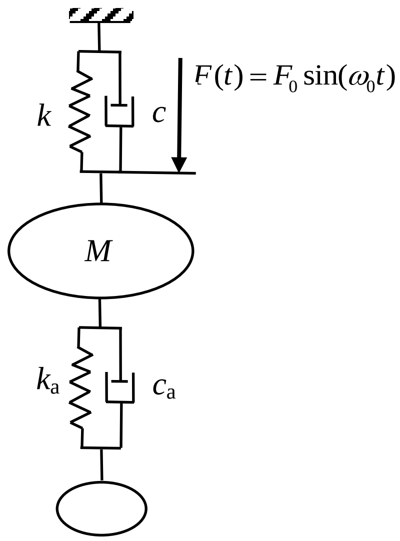

Fig. 1 – The mechanical system.

coefficient of viscous friction c, both attached to a mass M, as shown in Fig. 1.

This subsystem is called primary system. The second subsystem is represented by the vibration dynamic absorber which consists in a mass m, attached to mass M by a spring whose elastic constant is k_0 and damper having the coefficient of viscous friction c_a. The mass M is acted by a sinusoidal force, which, as input data, has the expression:

$$F(t) = F_0 \sin(\omega_0 t) \tag{1}$$

3. Construction of the Bond-Graph Model

The mechanical system pictured in Fig. 1 contains two masses, which implies two inertial elements (I) in the bond-graph model, one for each mass. A capacitive element (C) corresponds to each spring and a dissipative element (R) is adopted for each damper (Borutzky, 2010; Păstrăvanu and Ibănescu, 2001).

The force which acts upon mass M will be an ideal source of effort. Because the ends of the spring and damper between the two masses are mobile, a junction 0 is necessary for modelling the spring elongation, needed in the elastic force assessment, and the velocity, needed in the assessment of the force in the damper. For the velocity of each mass, a junction 1 is introduced.

The bond-graph model is presented in Fig. 2.

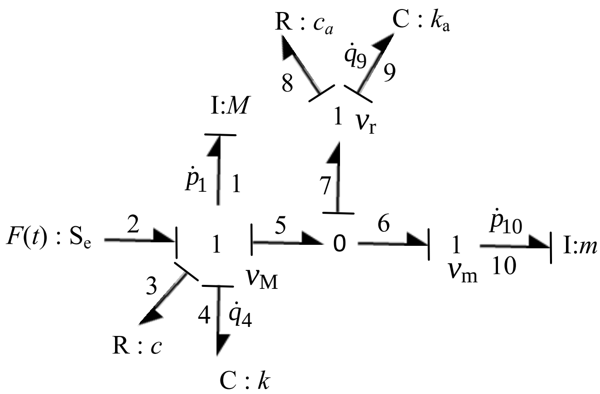

Fig. 2 – The bond graph model of the mechanical system.

4. The State-Space Model

The bond-graph model contains four energy storing elements in integral causality, that is two elements I, corresponding to the two masses and two elements C, corresponding to the two springs.

The state system of equations will have four differential linear equations with constant coefficients. The state variables are: p_1, p_{10}, q_4, and q_9.

The constitutive equations of junction 1, corresponding to mass M velocity, are:

$$-\dot{p}_1 + e_2 - e_3 - e_4 - e_5 = 0 \tag{2}$$

and

$$f_2 = f_3 = f_4 = f_5 = f_1 = \frac{p_1}{M} \tag{3}$$

where:

$$e_2 = F_0 \sin(\omega_0 t) \tag{4}$$

$$e_3 = c f_3 \tag{5}$$

$$e_4 = k\, q_4 \tag{6}$$

In order to express the effort e_5 in terms of state variables, the constitutive equations of junction 0 are expressed:

$$e_5 = e_6 = e_7 \tag{7}$$

$$f_5 - f_7 - f_6 = 0 \tag{8}$$

The equation of effort e_7 in terms of state variables is determined based on the constitutive equations of junction 1, corresponding to relative velocity v_r:

$$e_7 - e_8 - e_9 = 0 \tag{9}$$

$$f_8 = f_9 = f_7 \tag{10}$$

From Eq. (9) it results that:

$$e_7 = c_a f_8 + k_a q_9 \tag{11}$$

On the other side, the bond-graph model leads to the following equalities:

$$f_8 = f_7 = f_5 - f_6 = \frac{p_1}{M} - \frac{p_{10}}{m} \tag{12}$$

By using Eq. (4), (5), (6), (11) and (12), Eq. (2) becomes:

$$\dot{p}_1 = -\frac{c}{M} p_1 - k q_4 - c_a \left(\frac{p_1}{M} - \frac{p_{10}}{m} \right) - k_a q_9 + F_0 \sin(\omega_0 t) \tag{13}$$

from which the first state equation is obtained:

$$\dot{p}_1 = -\frac{c + c_a}{M} p_1 + \frac{c_a}{m} p_{10} - kq_4 - k_a q_9 + F_0 \sin(\omega_0 t) \tag{14}$$

The second state equation is determined by starting from the state equations of junction 1, corresponding to mass *m* velocity:

$$\dot{p}_{10} = e_6 \tag{15}$$

$$f_6 = f_{10} \tag{16}$$

By substituting Eq. (7) and (11) in Eq. (15), it results:

$$\dot{p}_{10} = c_a f_8 + k_a q_9 \tag{17}$$

Considering Eq. (12), the second state equation is obtained:

$$\dot{p}_{10} = c_a \frac{p_1}{M} - c_a \frac{p_{10}}{m} + k_a q_9 \tag{18}$$

The third state equation is derived by the relationship:

$$\dot{q}_4 = f_4 \tag{19}$$

which, by using Eq. (3), leads to the final form:

$$\dot{q}_4 = \frac{p_1}{M} \tag{20}$$

The fourth state equation is:

$$\dot{q}_9 = f_7 \tag{21}$$

which, by using Eq. (12), has the final form:

$$\dot{q}_9 = \frac{p_1}{M} - \frac{p_{10}}{m} \tag{22}$$

The four state-space equations could be solved by using different methods. In this paper, the diagram block provided by MATLAB-SIMULINK is considered, because it is very easy to use it.

5. The Integration of the State-Space Equations System by Using MATLAB-SIMULINK

In SIMULINK from MATLAB the state equations are integrated by using the block state-space (Karris, 2008)

For this reason, the system is written in the following matrix form:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{U}(t) \tag{23}$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{U}(t) \tag{24}$$

For the system of state equations generated by using the bond-graph method, the matrices in Eq. (23) and (24) are:

$$\mathbf{x}(t) = \begin{bmatrix} p_1 \\ p_{10} \\ q_4 \\ q_9 \end{bmatrix} \tag{25}$$

$$\mathbf{A} = \begin{bmatrix} -\dfrac{c+c_a}{M} & \dfrac{c_a}{m} & -k & -k_a \\ \dfrac{c_a}{M} & -\dfrac{c_a}{m} & 0 & k_a \\ \dfrac{1}{M} & 0 & 0 & 0 \\ \dfrac{1}{M} & -\dfrac{1}{m} & 0 & 0 \end{bmatrix} \tag{26}$$

$$\mathbf{B} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{27}$$

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \tag{28}$$

$$\mathbf{D} = 0 \tag{29}$$

The variable q_9 is the relative deformation of the absorber spring. The absolute motion of mass m is given by q_4-q_9, which explains the second row in matrix C in Eq. (28).

The matrices are symbolically written in the state-space block and the numerical values are written in a separate file for easy modifications.

In Fig. 3, the model introduced in SIMULINK is shown.

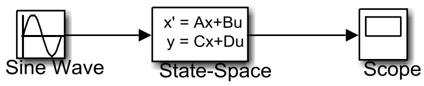

Fig. 3 – The model introduced in SIMULINK.

For the numerical simulation, the following values of the parameters have been considered: M = 3000 Kg, m = 200 Kg, c = 2000 Ns/m, c_a = 1000 Ns/m, k = 100000 N/m, k_a = 10000 N/m, F_0 = 0.1 N, $\omega_0 = 2\pi\,s^{-1}$. There are no initial conditions.

The results of the numerical simulation are presented in Fig. 4.

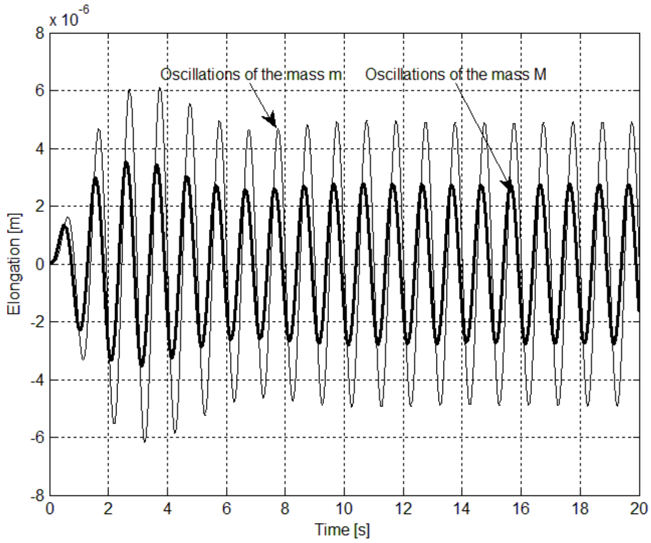

Fig. 4 – Oscillations of masses M and m.

6. Conclusions

The construction of the bond-graph model for the vibrating system provided with a dynamic absorber with viscous friction and its space-state equations represent a simpler way of analysis than other methods in system dynamics, as Lagrange equations.

The state equations can be quickly implemented in MATLAB – SIMULINK, this operation presuming to simply write some matrices in the state-space block.

The change of numerical values can be easily done, offering the possibility of studying the influence of each parameter, in order to optimize the dynamic absorber effect upon the mechanical system.

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MODELUL BOND-GRAPH AL UNUI SISTEM VIBRANT AMORTIZAT PREVĂZUT CU UN ABSORBITOR DINAMIC CU FRECARE VÂSCOASĂ

(Rezumat)

Vibrațiile provenite de la surse exterioare care acționează asupra sistemelor sunt de cele mai multe ori fenomene nedorite care conduc la disfuncționalități importante ale acestora. Atenuarea până la aproape de anulare a efectelor negative ale vibraților induse din exterior sistemelor se poate face în diverse moduri, unul dintre acestea fiind atașarea unui absorbitor dinamic de vibrații. Lucrarea prezintă obținerea modelului matematic prin metoda bond-graph al unui sistem vibrant amortizat supus acțiunii unei forțe sinusoidale, de care este atașat un absorbitor dinamic cu frecare vâscoasă. Din modelul bond-graph sunt obținute ecuațiile de stare care sunt integrate folosind mediul MATLAB-SIMULINK. Modelul poate fi utilizat ulterior pentru a studia eficiența absorbitorului în funcție de variația parametrilor săi.